External Fields and the

Dynamics of Flavours in Holographic Duals of Large N Gauge Theories

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Based on work with
Tameem Albash, Veselin G. Filev, Clifford V. Johnson
hep-th/0605088, arxiv:0709.1547, arxiv:0709.1554, arxiv:0803.0038
+ work in progress ...

Strings & Gauge Theories Workshop, MCTP 2008

Motivation & Outline

- Strongly coupled large N gauge theory with flavours

  dynamics of fundamental particles, e.g. quarks

  response to external parameters, phase diagram

  seek at least qualitative universal features

- Flavour branes in AdS-background

  probing by D7-brane

  the meson melting phase transition

  dynamics of flavours in external field, the phase structure

- Play the same game elsewhere

The type IIB SUGRA Model

String theory on $AdS_5 \times S^5$

isometry: $SO(4,2)$, $SO(6)$

Addition of N_f D7-brane

$SO(6) \to SO(4) \times SO(2)$

breaking of $SO(2)$

AdS-BH geometry

$\mathcal{N} = 4$ super Yang-Mills

superconformal, global R-symmetry

Adjoint fields: $A_\mu, 4\lambda, 6\phi$

Addition of $\mathcal{N} = 2$ hypermultiplet

$SO(2) \simeq U(1)$

chiral symmetry

breaking of $U(1)$ phase rotation of the flavours

finite temperature (broken SUSY)

AdS-Schwarzschild:

$$ds^2 = -\frac{f(u)}{R^2}dt^2 + \frac{R^2}{f(u)}du^2 + \frac{u^2}{R^2}d\vec{x}\cdot d\vec{x} + R^2\left(d\theta^2 + \cos^2\theta d\Omega_3^2 + \sin^2\theta d\phi^2\right)$$

$$f(u) = u^2 - \frac{b^4}{u^2}\ , \quad b^2 = \frac{8G_5 m_{\rm bh}}{3\pi}$$

Euclideanize: $\Longrightarrow$ $T = \frac{b}{\pi R^2}$

Embedding ansatz: $\phi = 0;\ \theta = \theta(u)\ , \quad L = u\sin\theta$

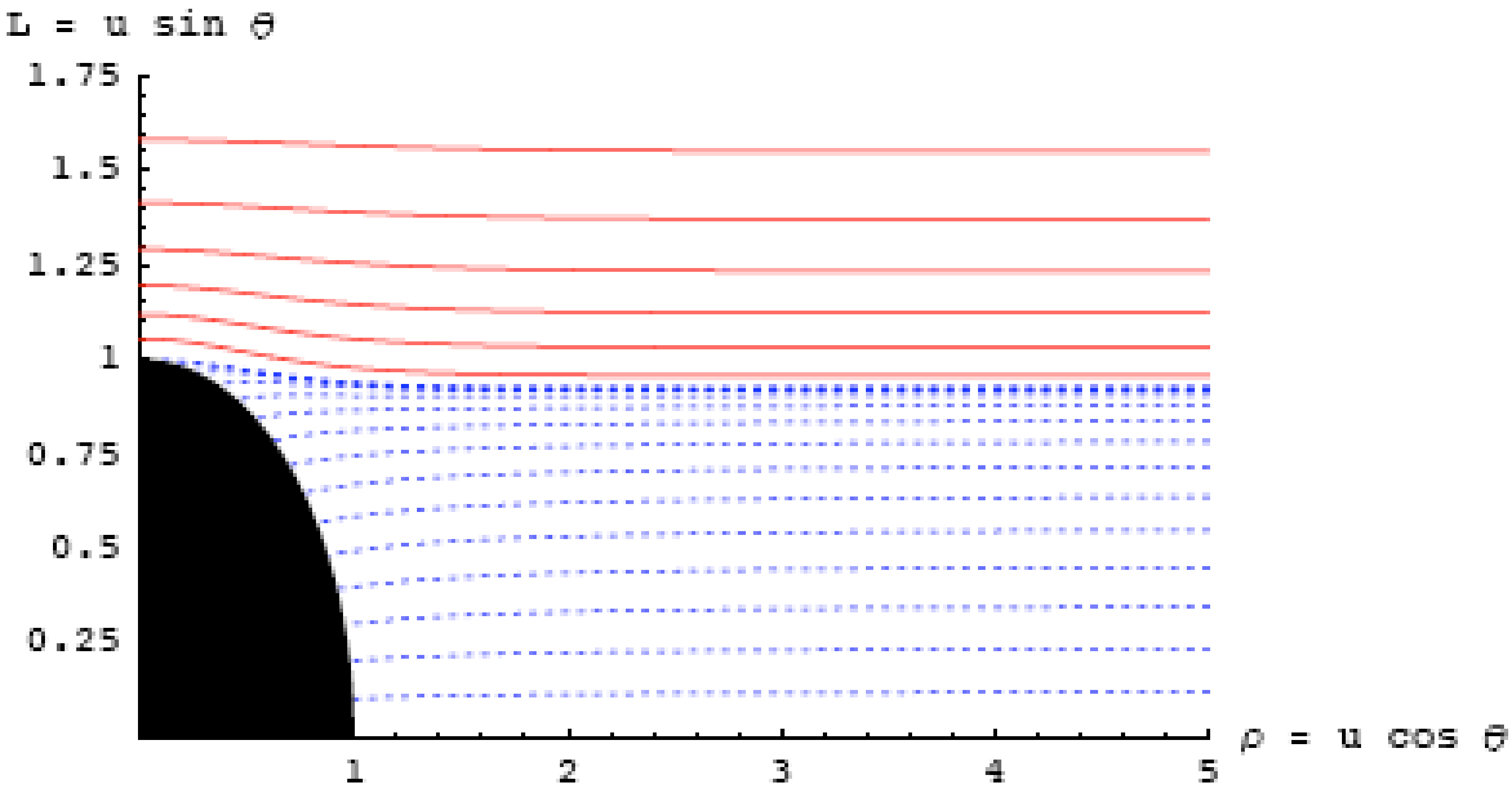
Pulled back D7 metric:

$$ds^2 = \frac{f(u)}{R^2}d\tau^2 + \frac{R^2}{f(u)}du^2 + \frac{u^2}{R^2}d\vec{x}\cdot\vec{x} + R^2\left(\frac{u^2 - L^2}{u^2}\right)d\Omega_3^2$$

contractible S^1 contractible S^3

The Embedding Solutions

Two kinds: blackhole and Minkowski



Babbington et. al, Myers et. al, Karch et. al, Albash et. al

The Phase Transition

At finite temperature the D7-brane has topology $\mathbb{R}^4 \times S^3 \times S^1$

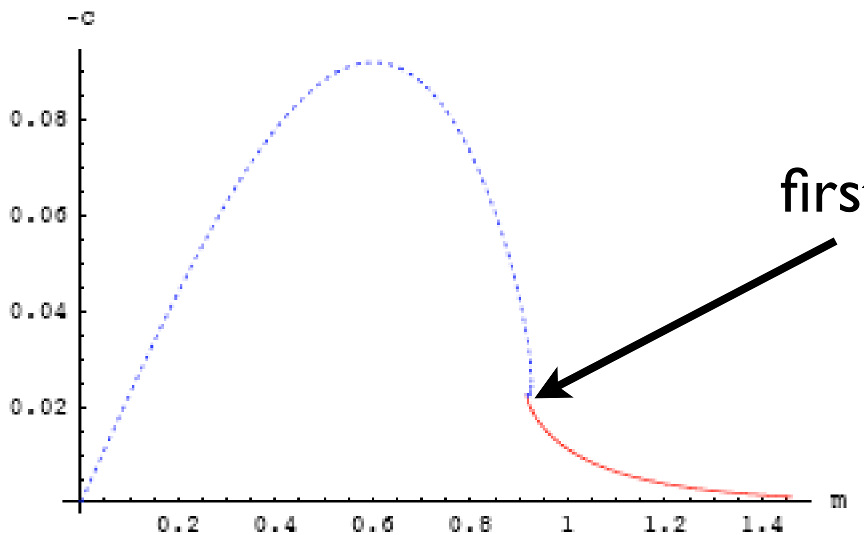
$S^3 \subset S^5$; S^1: euclidean time

Competition between S^3 and S^1 $\Longrightarrow$ phase transition

Embedding of D7 described by $L(u)$

$$\lim_{u\to\infty} L(u) = m + \frac{c}{u^3} + \dots$$

m: quark mass; c: condensate



first order phase transition at a critical value of bare quark mass

Babbington et. al, Myers et. al, Karch et. al, Albash et. al

Key Features of the Transition

- We are always in the deconfined phase

low T: probes end before the horizon

high T: probes fall into the blackhole

- Study of meson spectrum for the two cases

Minkowski embeddings: discrete spectrum of stable mesons
blackhole embeddings: existence of quasinormal frequency,
meson melting

Babbington et. al, Myers et. al, Karch et. al, Hoyos et. al, Albash et. al and many others ...

Introducing External Fields

Add a pure gauge B-field to AdS background

Filev et. al

$$B_{(2)} = H dx^2 \wedge dx^3$$

external magnetic field

The background does not change

The probe brane couples thru' the DBI action $B_{ab} + 2\pi\alpha' F_{ab}$

Equivalent to excite a gauge field on the probe worldvolume

$$A_2 = H x^3$$

The Fate of the Embeddings

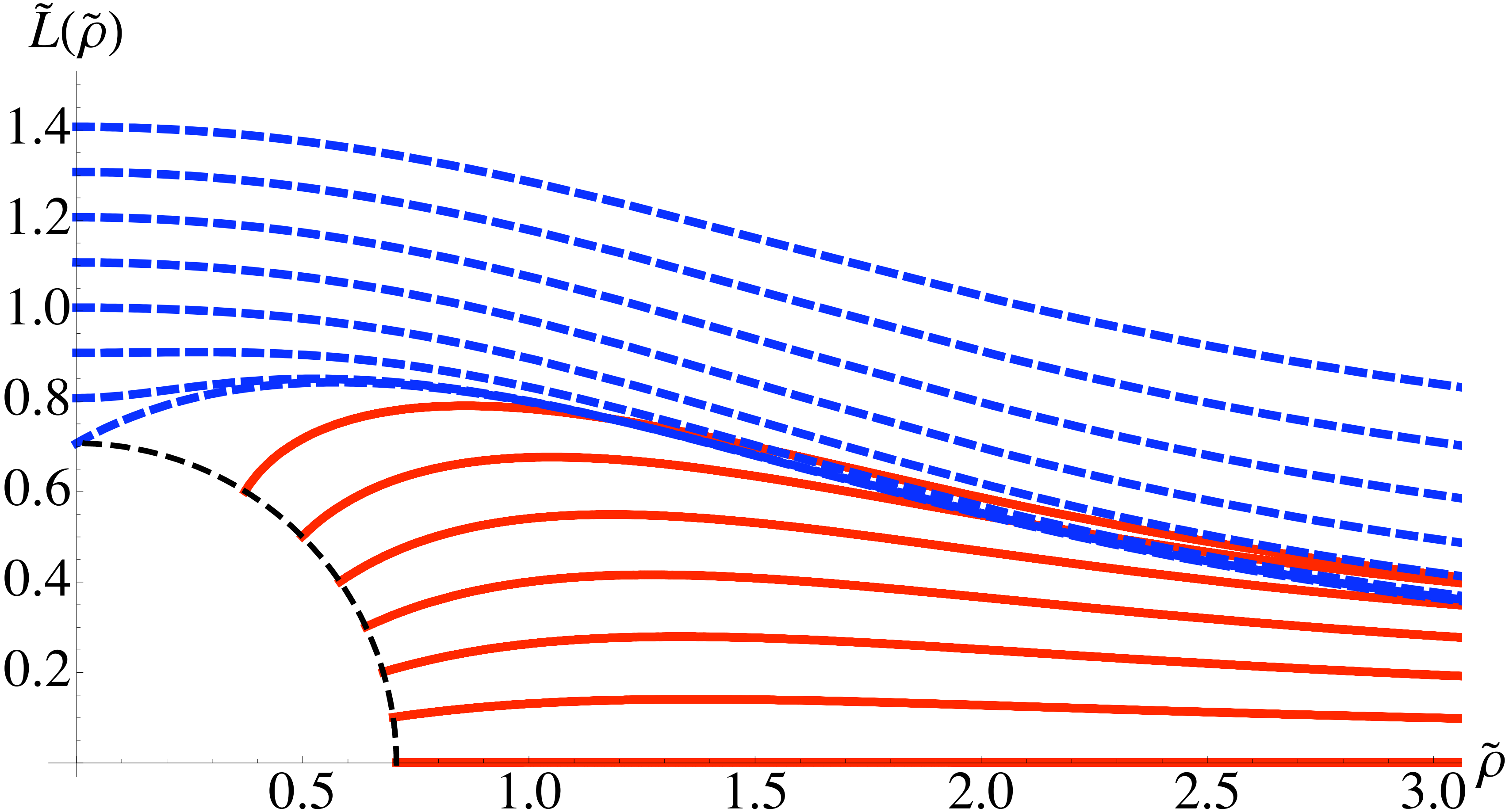
Useful variable changes:

$$r^2 = \frac{1}{2}\left(u^2 + \sqrt{u^4 - b^4}\right) = \rho^2 + L^2$$

$$\rho = r\cos\theta\ , \quad L = r\sin\theta$$

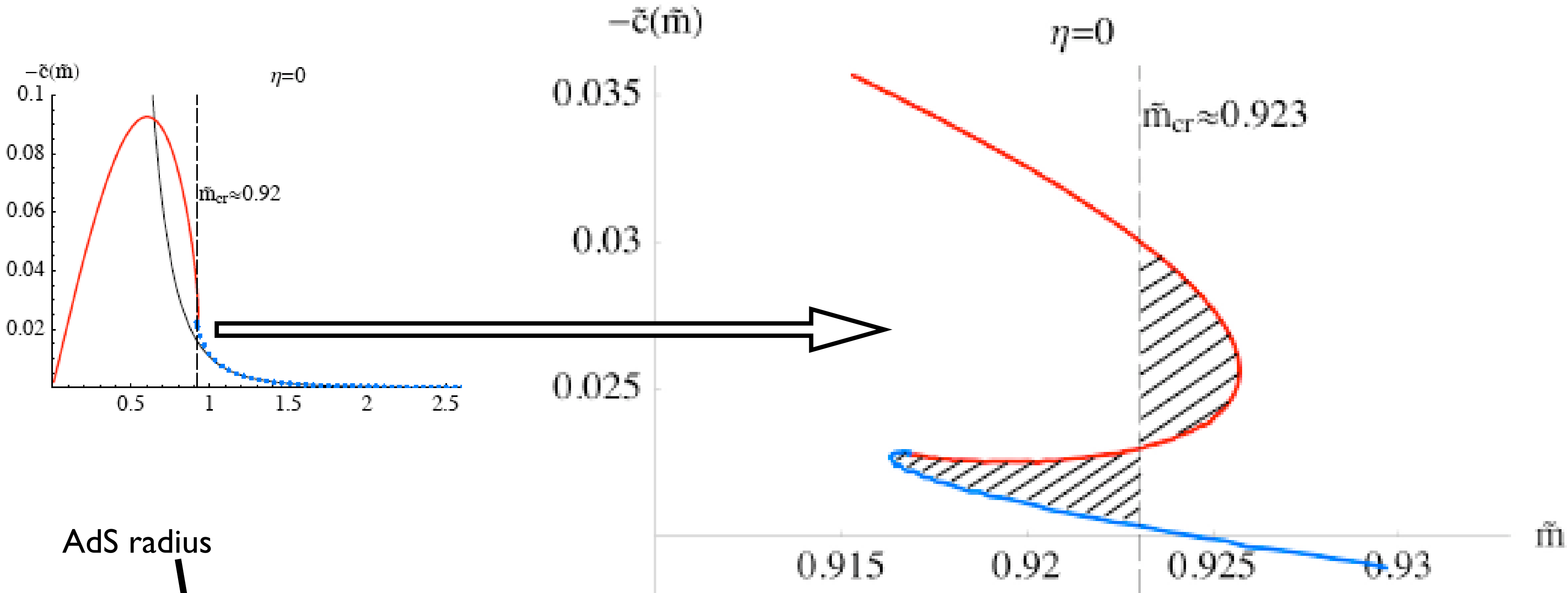
Dimensionless variables introduced:

$$\tilde{L} = L/b\ , \quad \tilde{\rho} = \rho/b$$



Back to the Melting Transition

Before switching on the magnetic field



AdS radius

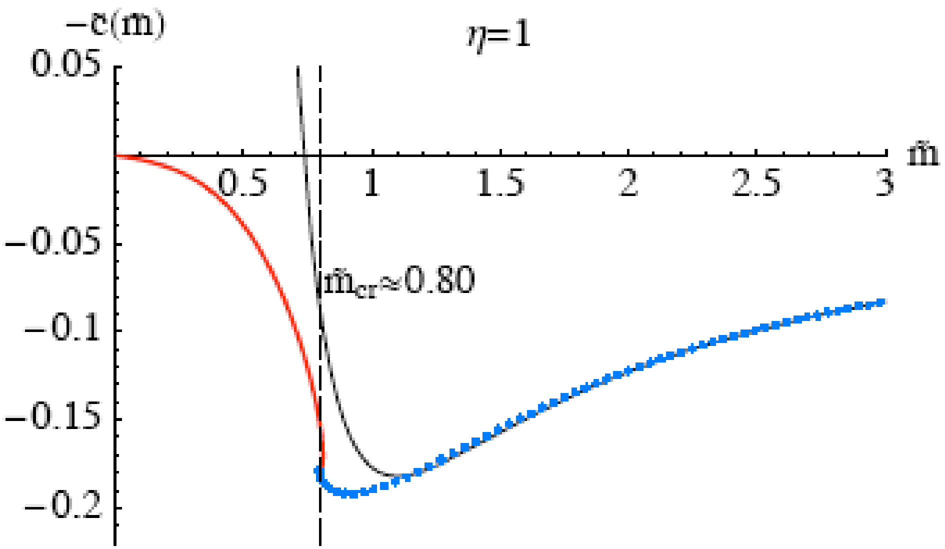
$$\eta = \frac{R^2}{b^2} H$$

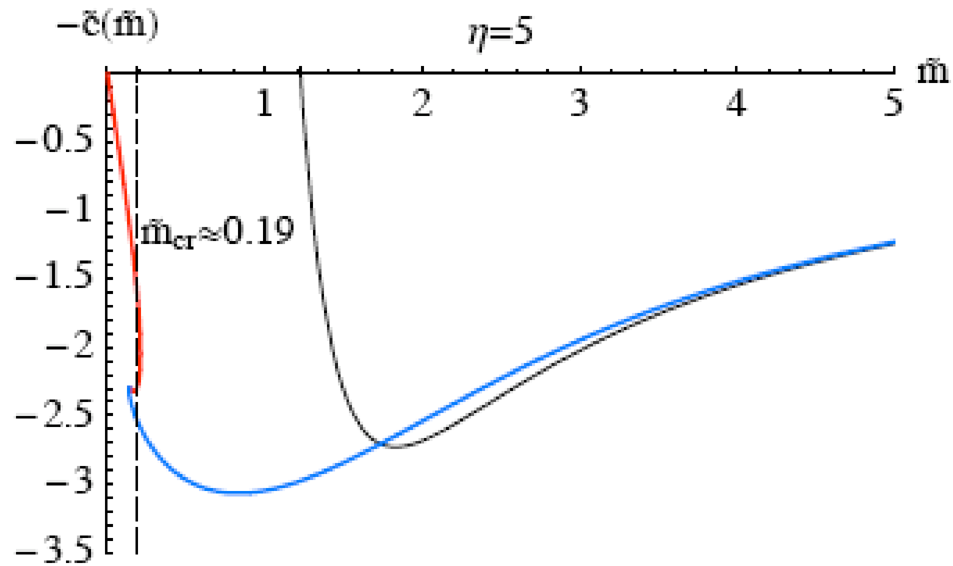
$$\tilde{m} = \frac{m}{b}$$

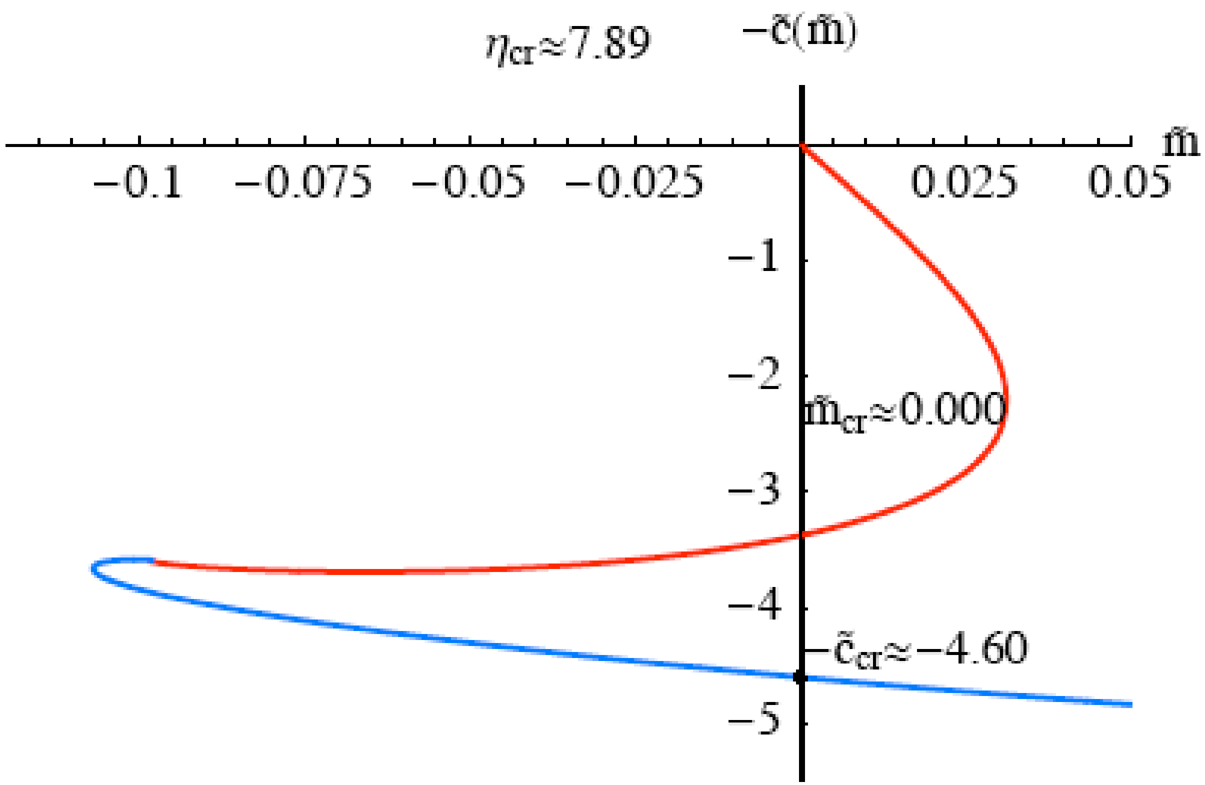
horizon

Now dial up the knob ...

Looking at the Transition







Only Minkowski embeddings are available

Key Features of the Transition

- Magnetic field competes with the temperature

  beyond the critical field there is no melting transition

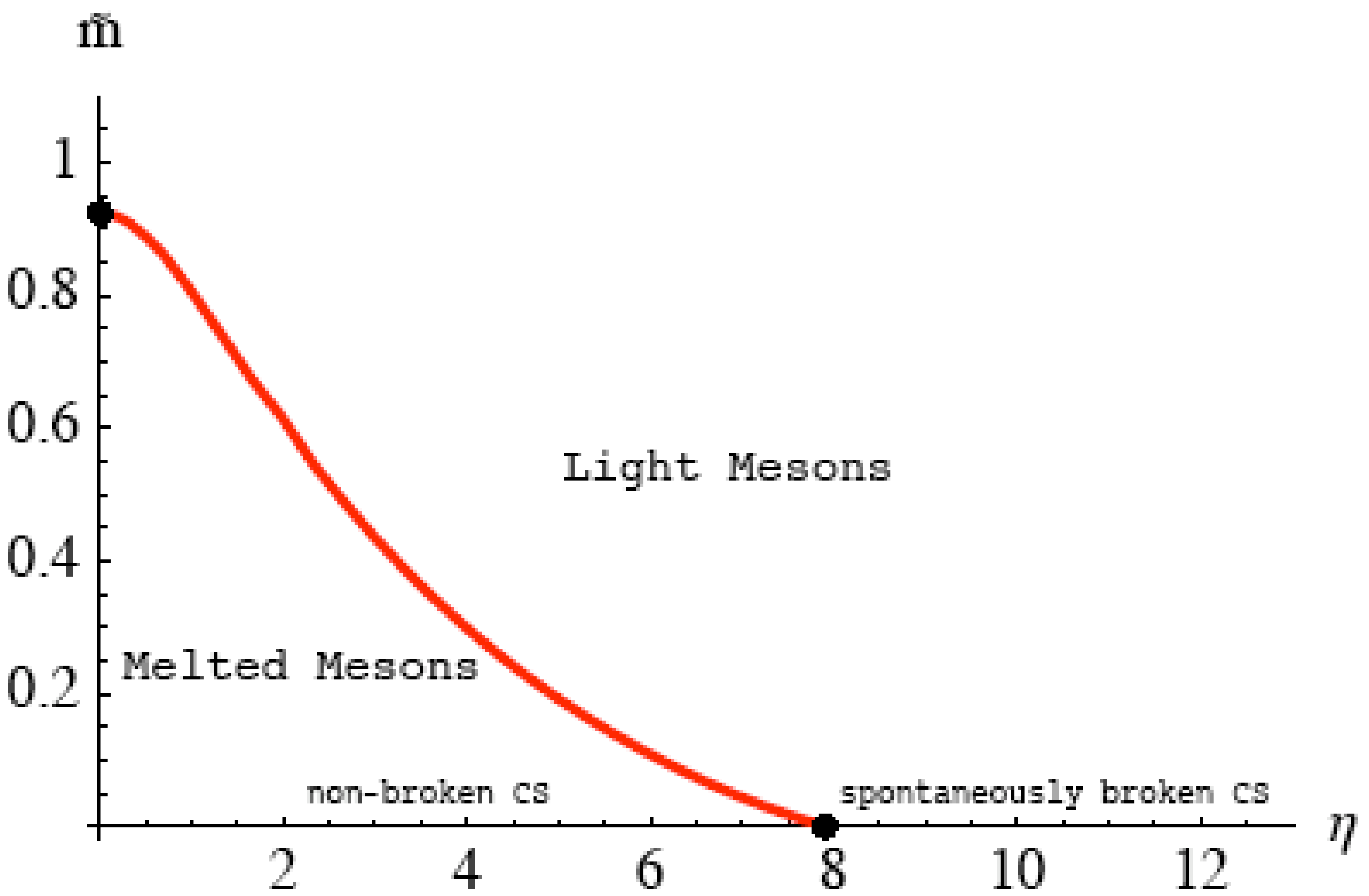
  chiral symmetry is spontaneously broken

Albash et. al, Erdmenger et. al

- At high enough magnetic field we recover zero temperature behaviour

Filev et. al

Summarising the Phase Structure



$$\eta = \frac{R^2}{b^2} H, \quad \tilde{m} = \frac{m}{b}$$

Albash et. al

Introducing an Electric Field

Consider exciting appropriate gauge field on the world-volume of the probe

Karch & O' Bannon

$$A_1(r) = -Et + B(r)$$

external electric field

Also turns on a current, which is the normalisable mode of $B(r)$

$$\lim_{\tilde{r}\to\infty} B(r) = \frac{\tilde{T}}{2\tilde{r}^2} + \ldots \ , \qquad \langle J^1 \rangle \propto \tilde{T}$$

Useful variable changes:

$$r^2 = \frac{1}{2}\left(u^2 + \sqrt{u^4 - b^4}\right) = \rho^2 + L^2$$

$$\rho = r\cos\theta \ , \qquad L = r\sin\theta$$

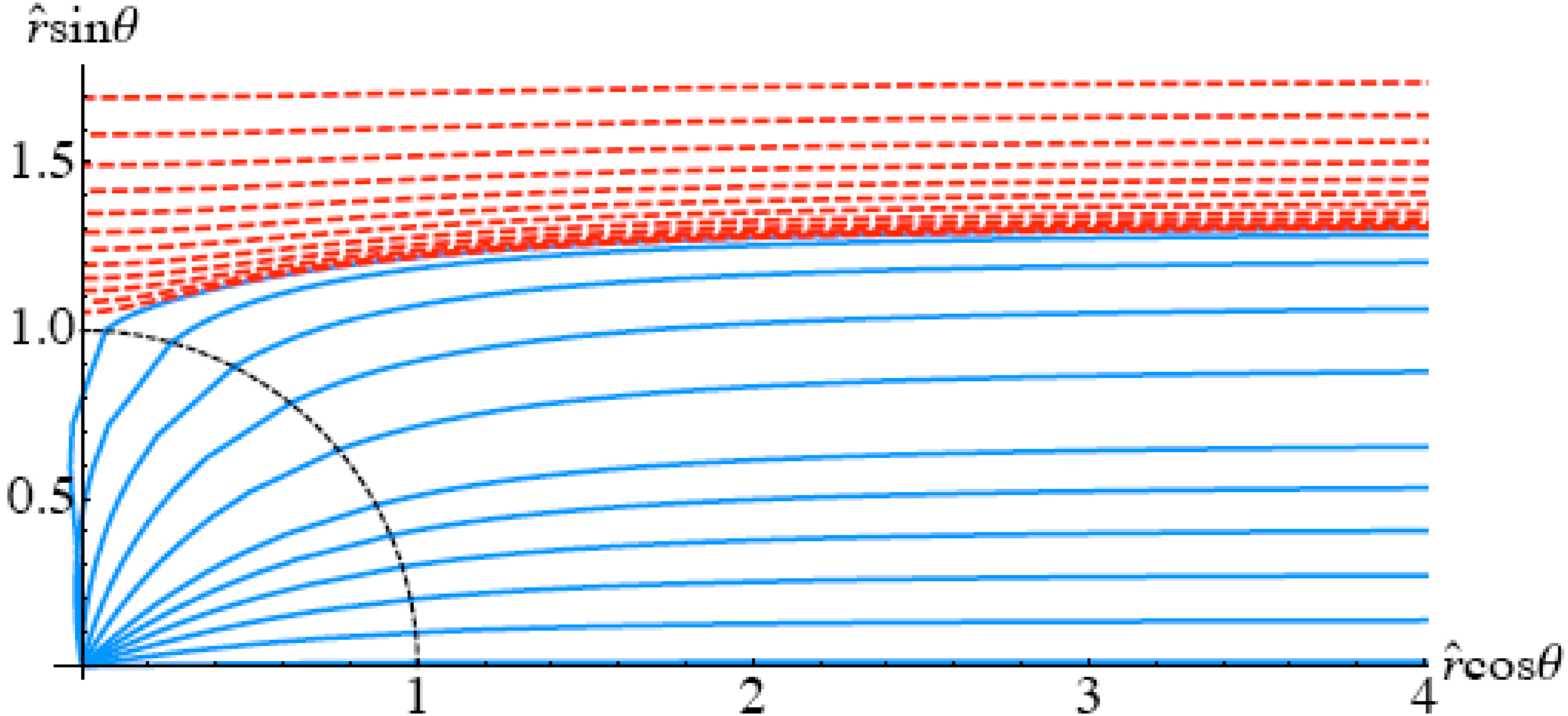
$$r = b\tilde{r} \qquad B(r) = \frac{b}{2\pi\alpha'}\tilde{B}(\tilde{r})$$

$$E = \frac{b}{2\pi\alpha' R^2}\tilde{E} \qquad \theta(r) = \tilde{\theta}(\tilde{r})$$

The Fate of the Embeddings

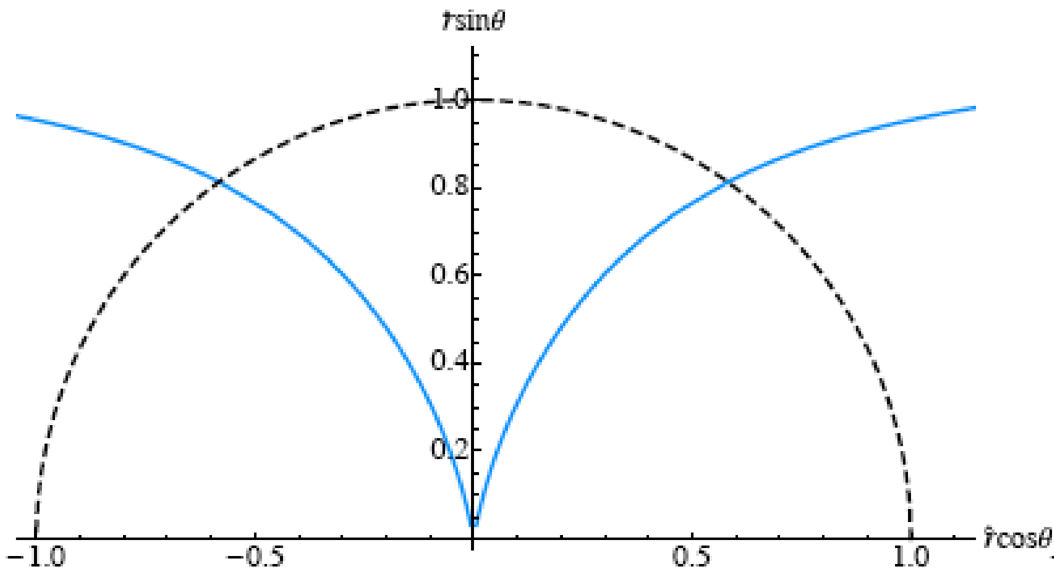
The case of vanishing temperature: different types of embeddings exist

Dimensionless variables introduced $r = R\sqrt{E}\hat{r}\ , \quad m = R\sqrt{E}\hat{m}$

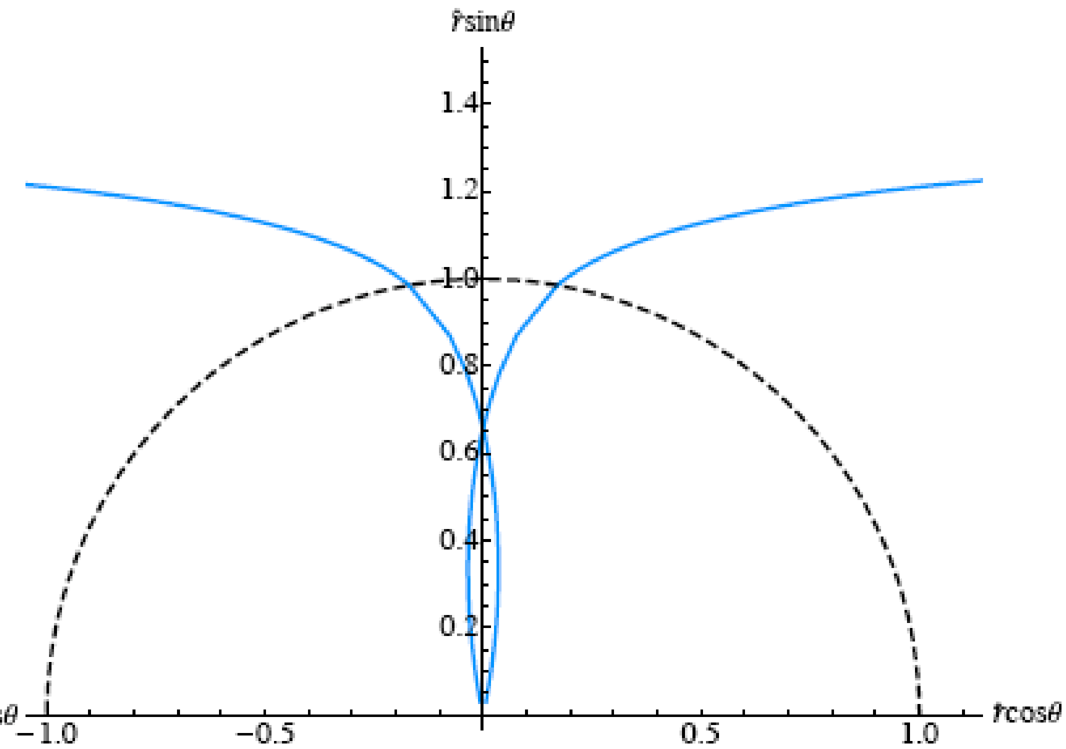


Albash et. al, Erdmenger et al

Three Different Types



(i) smooth

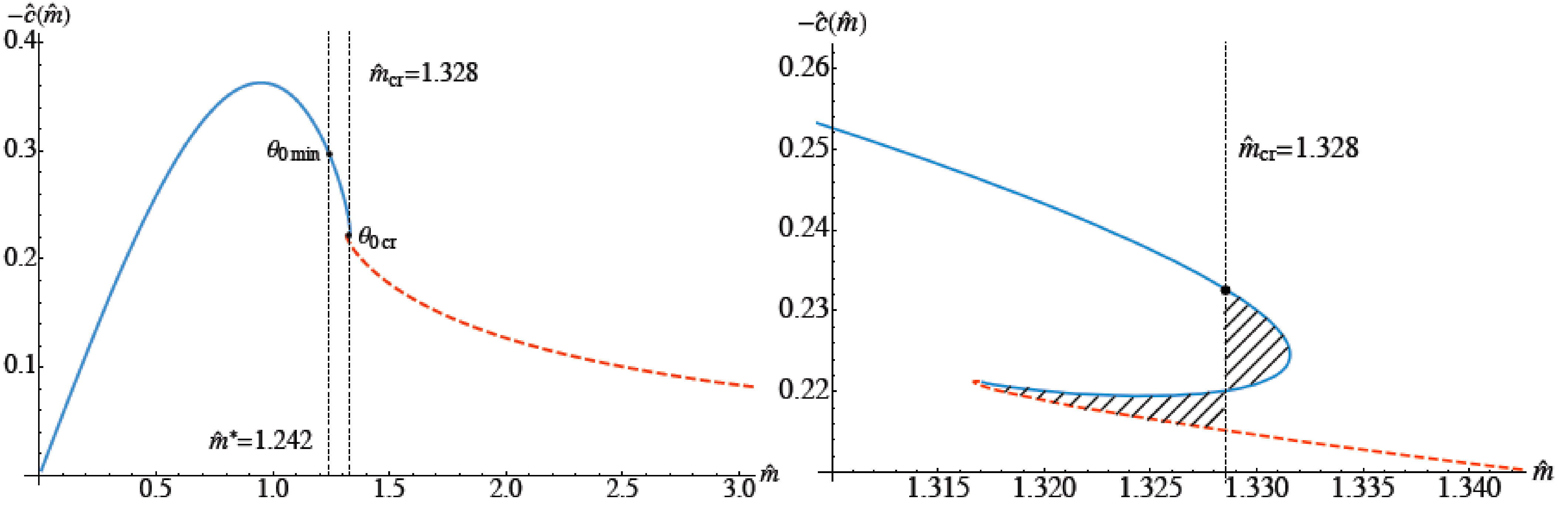


(ii) singular (excess angle)

(iii) Minkowski embeddings are present as before

we look at the behaviour of the condensate ...

Looking at the Phase Transition



There exists a transition mediated by the electric field even at zero temperature

Albash et. al, Erdmenger et. al

Key Features of the Transition

Minkowski embedding at large bare quark mass: bound mesons and vanishing current

↓

Non-Minkowski embedding at small bare quark mass: quarks liberated and non-zero current

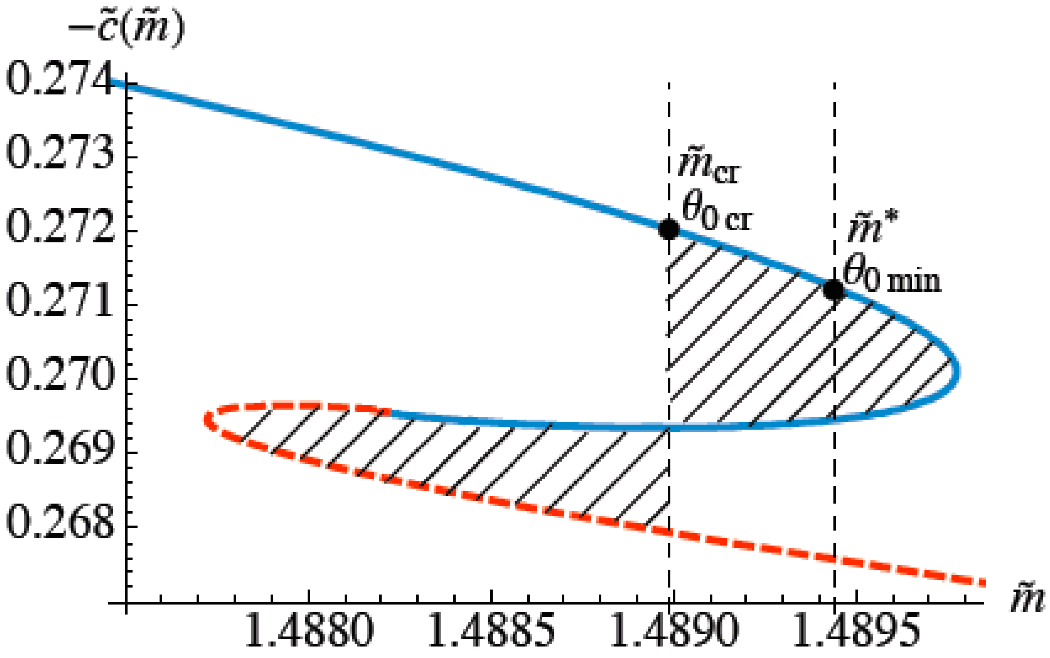
A strong electric field dissociates the mesons into constituent quarks and drives the current

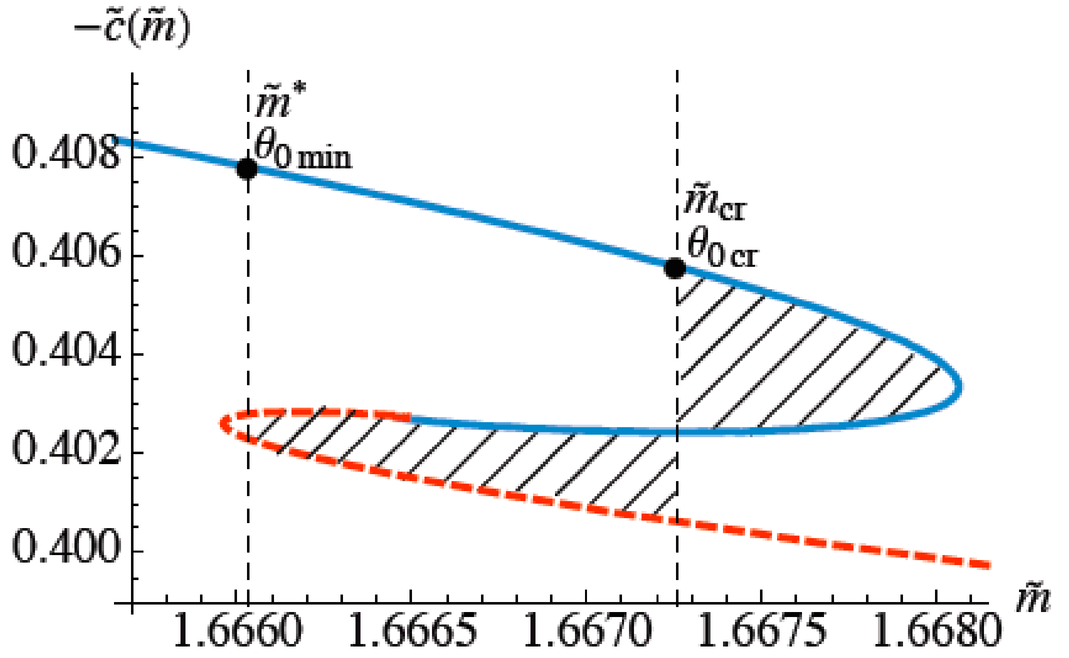
Caveat: the presence of singular solutions is not well-understood

Cranking up the Temperature

Embeddings can be classified into three different categories as before

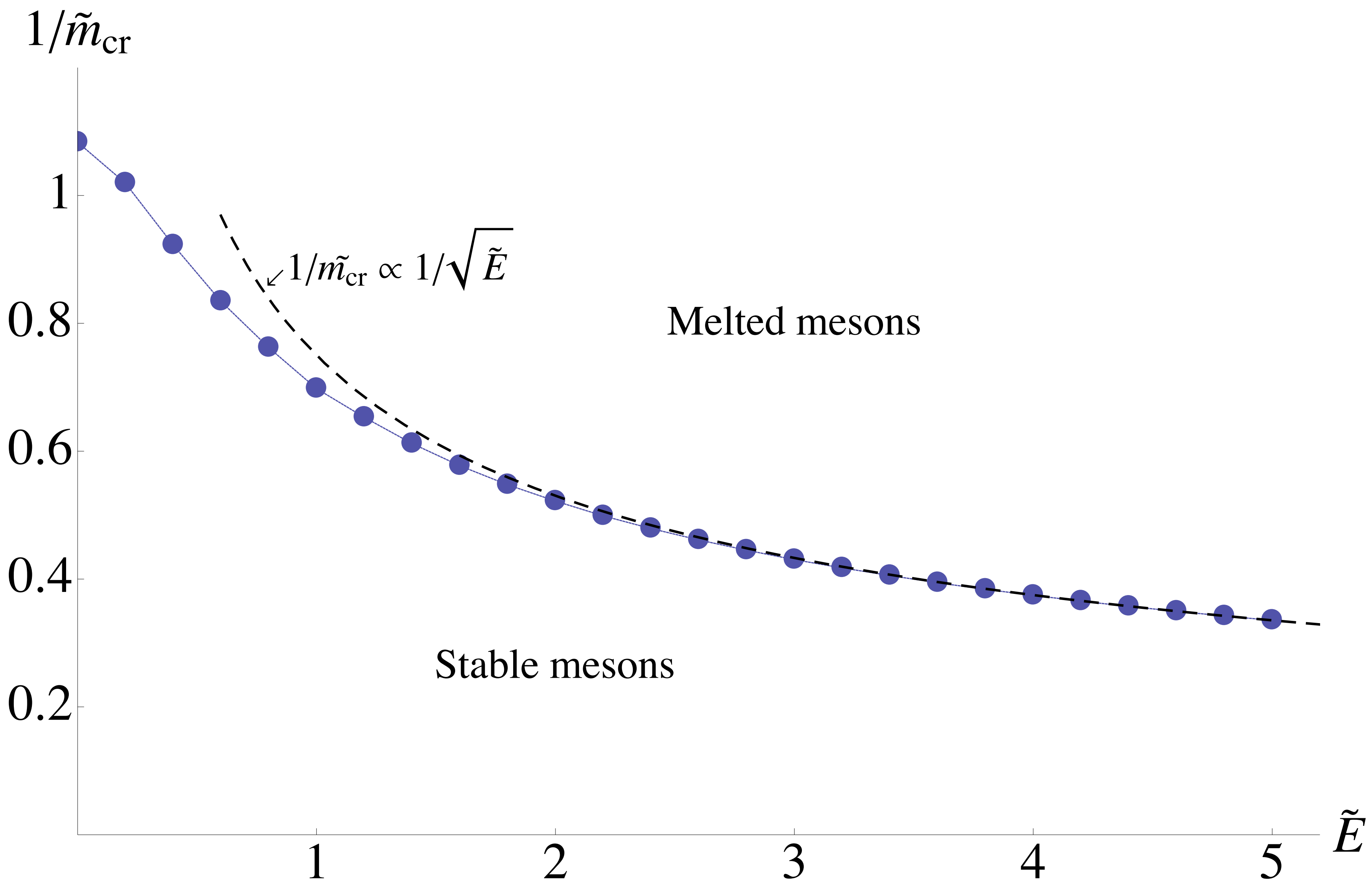
There exists a similar phase transition





At non-zero temperature the singular solutions can be bypassed by the dissociation temperature

Summarising the Phase Structure



The type IIA SUGRA Model (the Sakai-Sugimoto model)

N_c D4-branes wrapped on a circle	(4+1)-dim $SU(N_c)$ Yang-Mills
Addition of N_f D8-brane-- antibrane with $N_f \ll N_c$	global $U(N_f)_L \times U(N_f)_R$ symmetry (chiral symmetry in massless QCD)
Colour and flavour branes intersect	No bare quark mass
Flavour branes localized on the circle	quarks live in (3+1)-dim

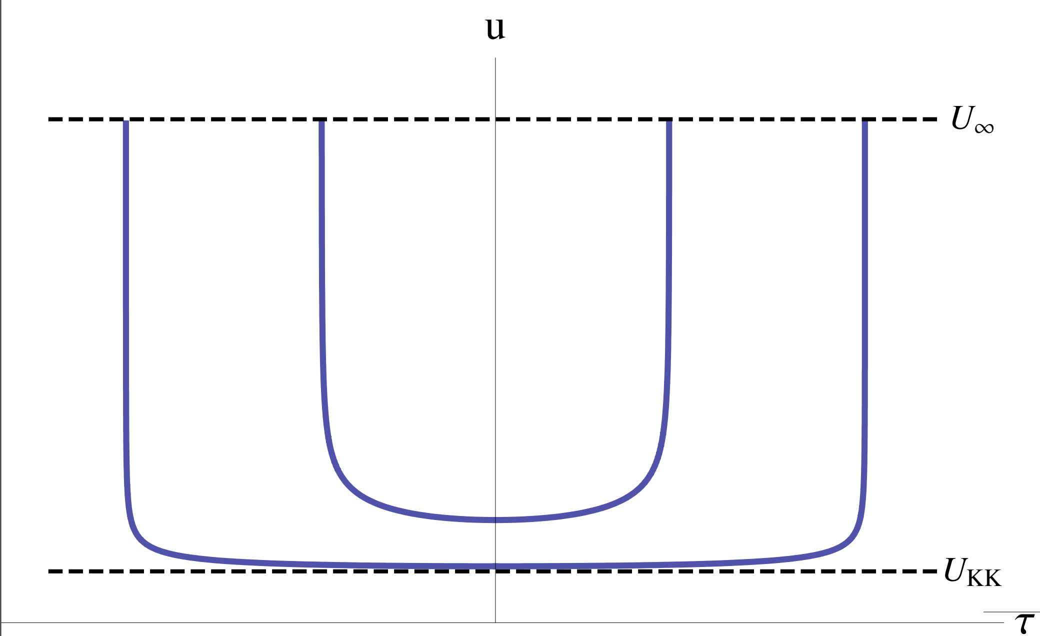
The Embedding Solutions

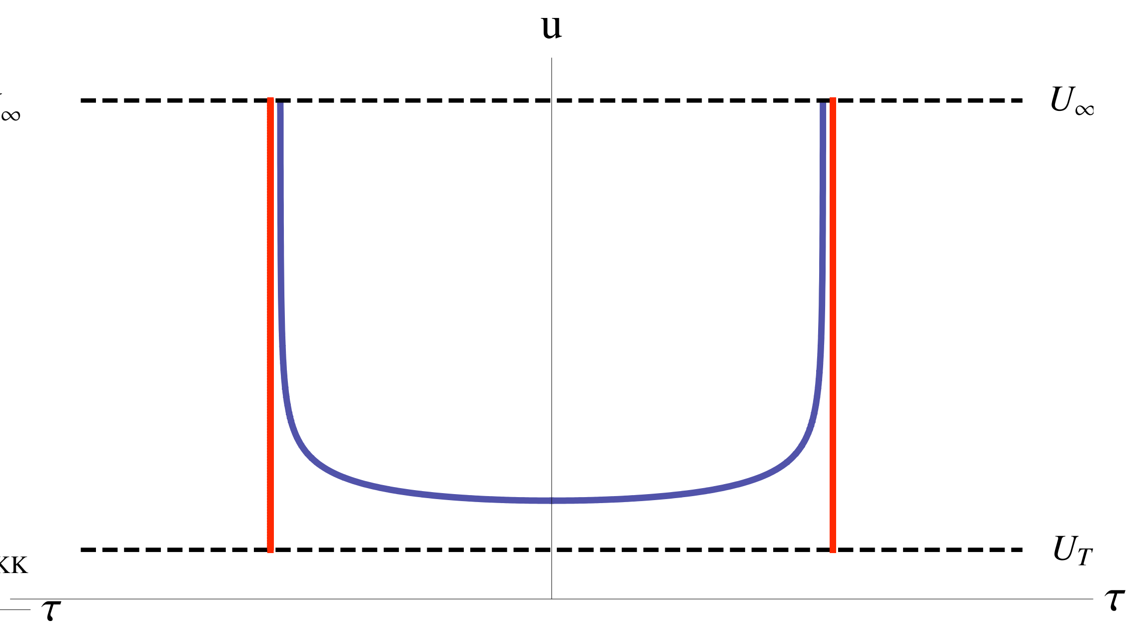
Low temperature phase: background geometry has cigar shape in $\{u, \tau\}$- plane

High temperature phase: background geometry has cigar shape in $\{u, t_E\}$- plane

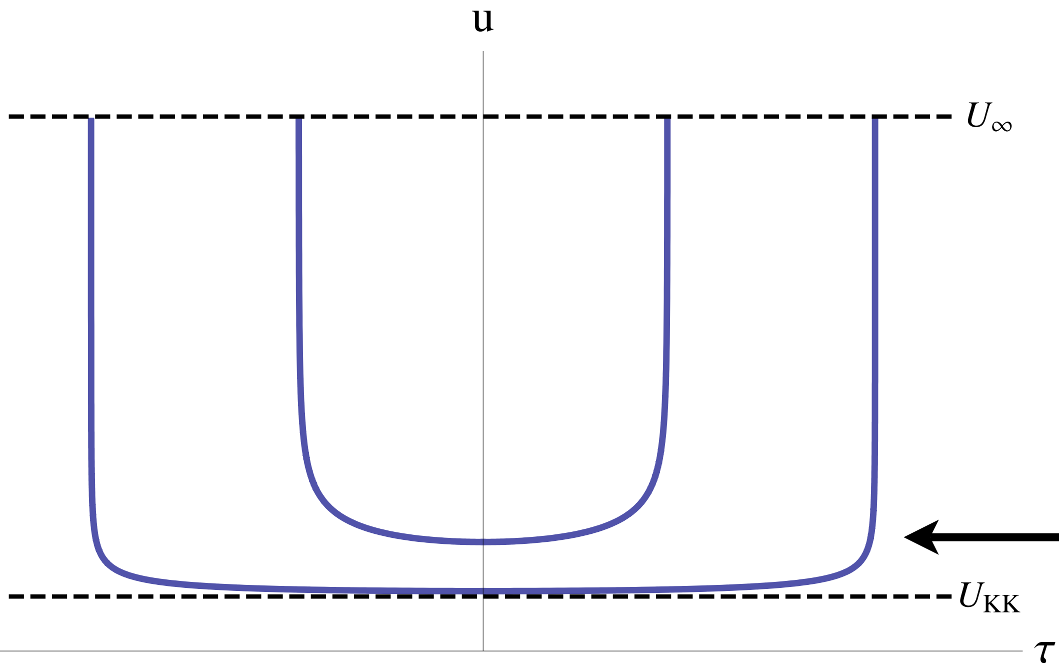
radius of the spatial circle, $R \sim U_{KK}^{-1/2}$

background temperature, $T \sim U_T^{-1/2}$





Pattern of Chiral Symmetry Breaking

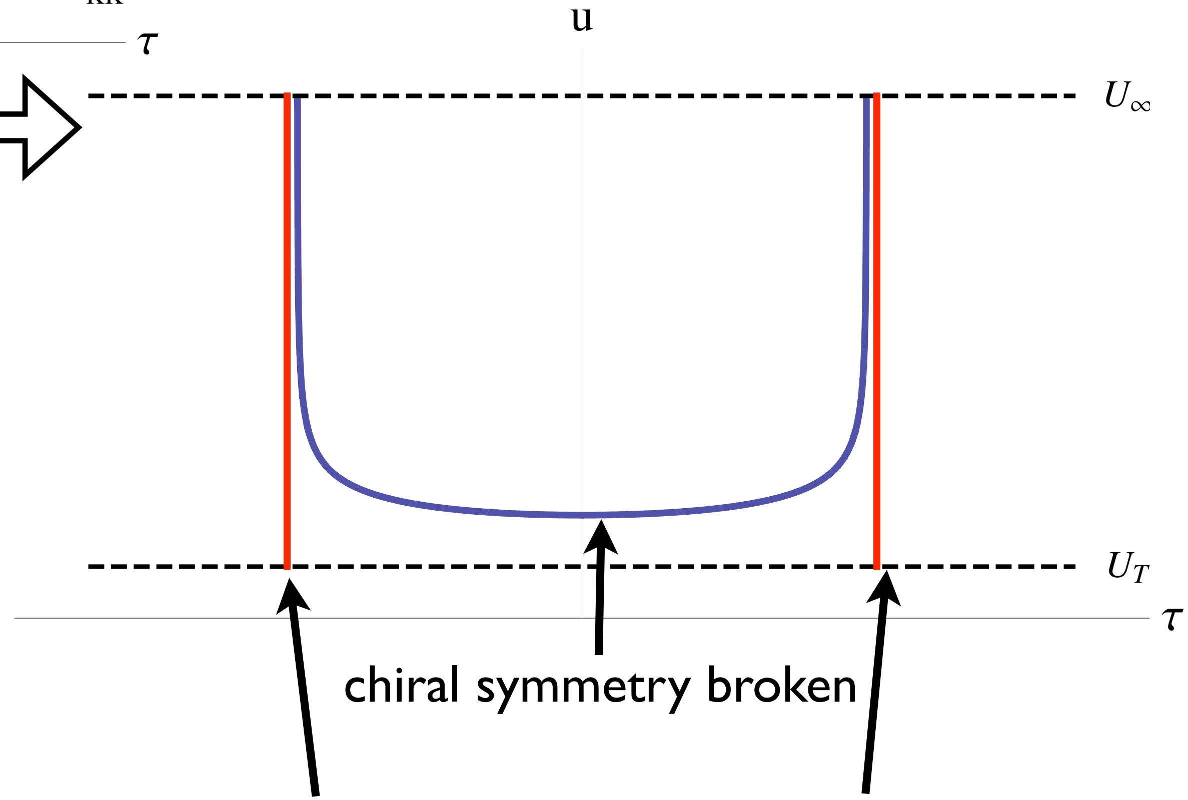


Low temperature background
(confined)

τ : compact spatial direction

$$U(N_f)_L \times U(N_f)_R \to U(N_f)_{diag}$$

High temperature background
(deconfined)



First order phase transition:

$$\boxed{T_{cr} \sim \frac{1}{L}}$$

External Fields Again

As before: $$B_{(2)} = H dx^2 \wedge dx^3$$

Confined or low temperature phase: trivial, chiral symmetry always broken topology rules, and magnetic field helps

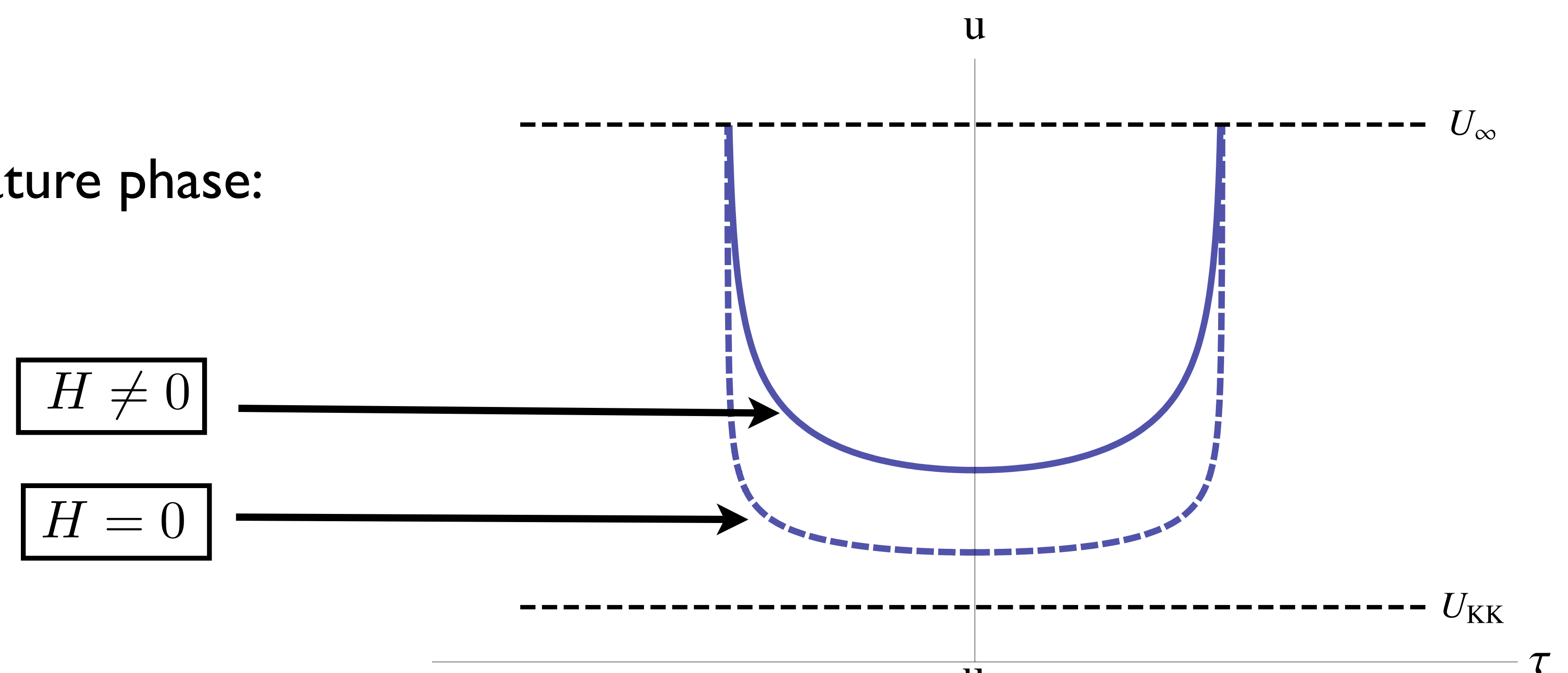
Deconfined or high temperature phase: non-trivial, temperature and magnetic field competes

Phase diagram given by: $$T_{cr} \sim f(H) \frac{1}{L}$$

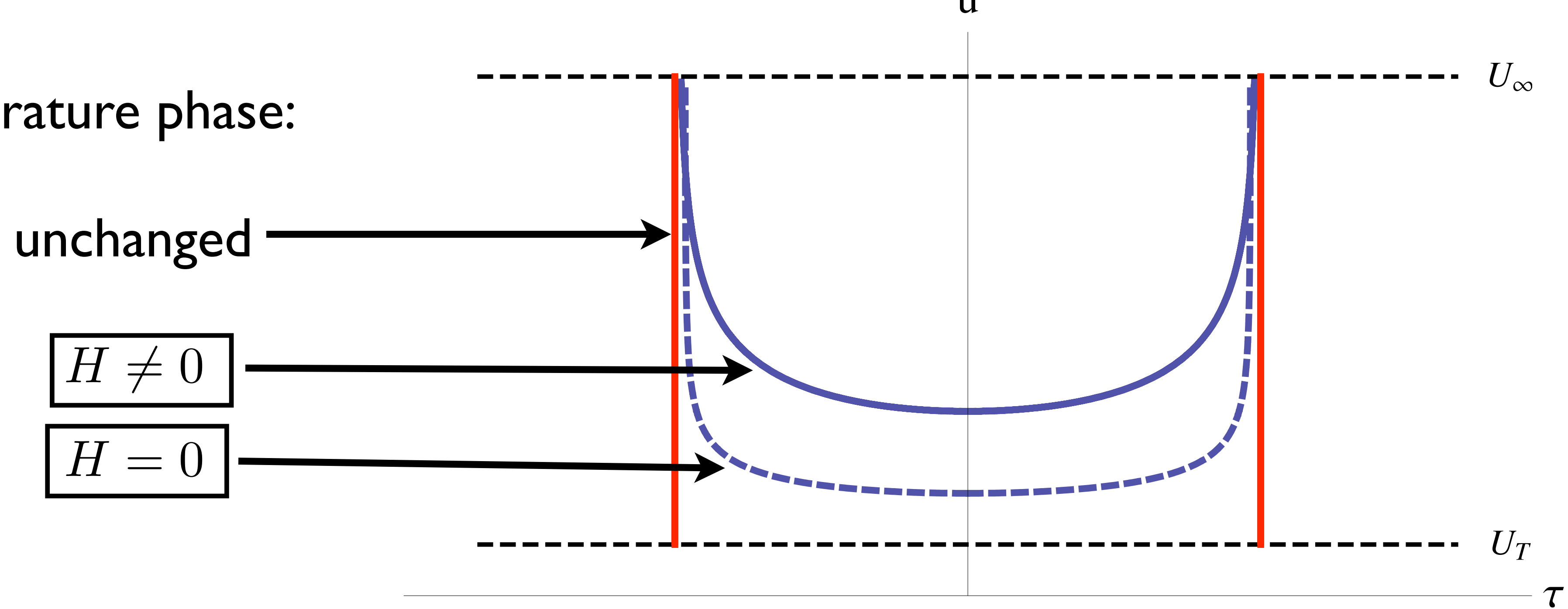
Expectation:
critical temperature should increase

The Fate of the Embeddings

Low temperature phase:



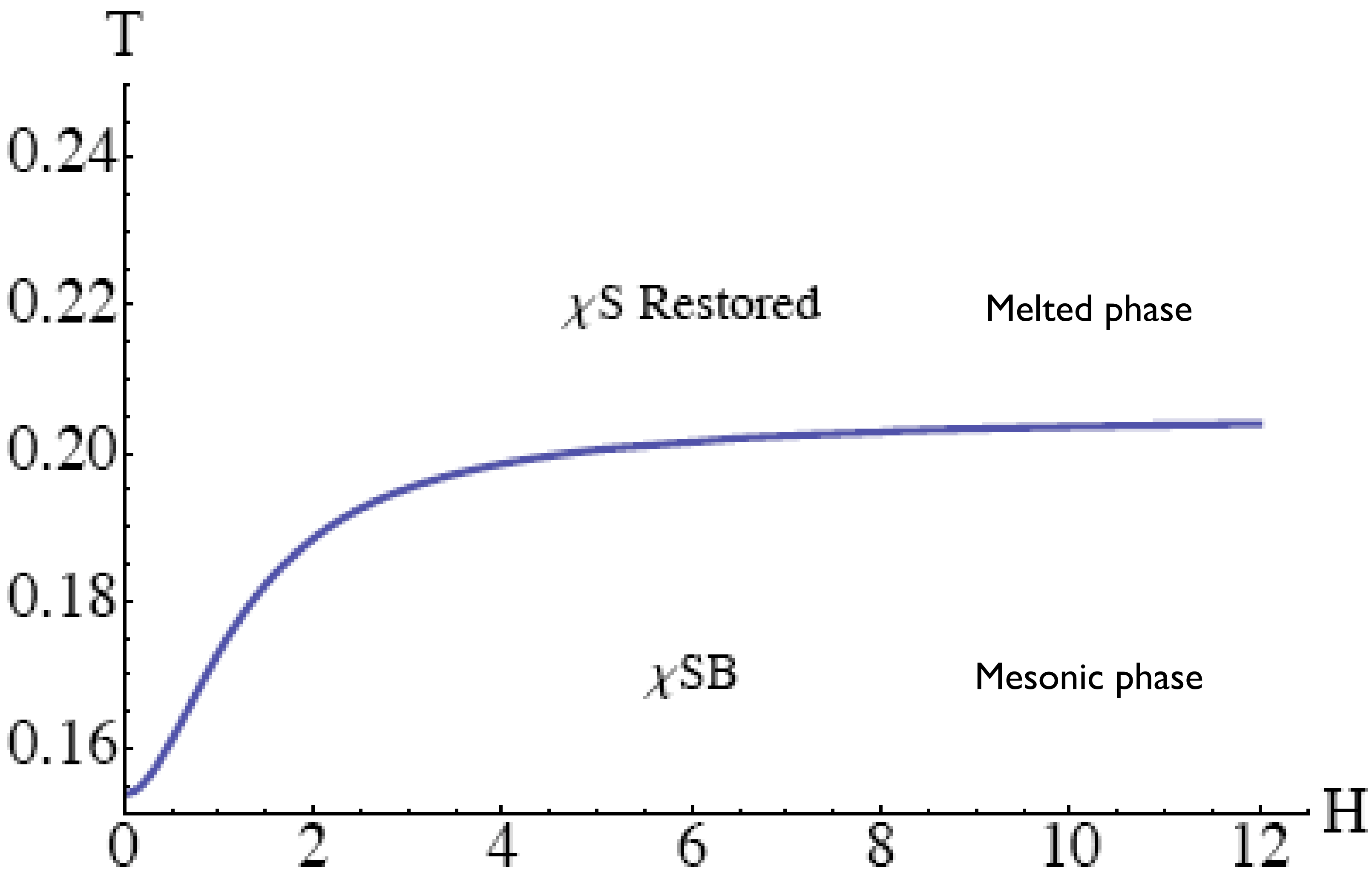
High temperature phase:



The Phase Structure

Energy decides the favoured embedding

Critical surface is described by: $\Delta S = 0$



Bergman et. al, Johnson et. al

Summary and Conclusion

- Phase structure is interesting in an external field

  magnetic catalysis in chiral symmetry breaking

  electric field drives a flavour current; hence conductivity

  interesting meson spectrum, thermodynamic observables

- Always more to do

  an universal understanding from geometry

  revise the singular solutions

  going beyond the probe limit...

  a more realistic model of QCD...

Thank you!!